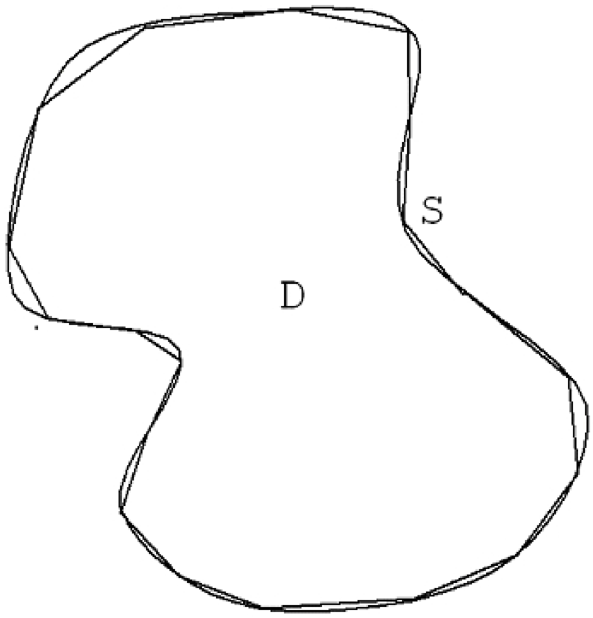
Boundary Representation in the Boundary Element Method

The boundary element method is a technique for solving a *boundary value problem*[1]*;* determining the solution of a problem in s field or domain from a given condition on the boundary (or boundaries) of that domain. The boundary element method can be a versatile method only if it includes the ability to represent any boundary in the given class of two-, three- or axisymmetric three-dimensional space. In general this is carried out by the facility of defining the boundary as a set of panels, each having the same characteristic form (or a set of two or three characteristic forms in more advanced software). For example a closed two- dimensional boundary can be represented by a set of straight lines, as illustrated in the following figure.



In the traditional boundary element method we typically modelled *closed* boundaries like the one illustrated above. However, techniques have arisen for modelling open boundaries or discontinuities and these may also be represented in a similar way.

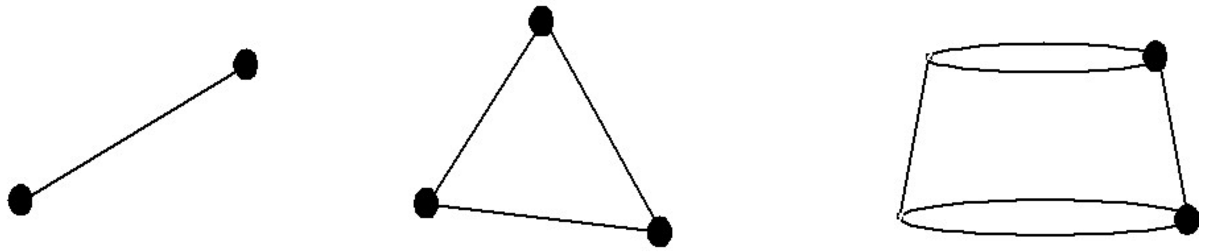
Let S be the original boundary and let $\Delta\tilde{S}_j$(for j = 1, 2, .., n) be the panels that represent an approximation to S in the boundary element method. If $\widetilde{S} = \sum_{j=1}^{n} \Delta\tilde{S}_j$ is the surface described by the complete set of panels then $\widetilde{S}$ is the approximation to S; $\widetilde{S} \approx S$. The representation of the boundary in this way is the first step in the discretisation of the integral operators that occur in the boundary integral formulations. Users of the boundary element method will focus mainly on the creation of the boundary mesh and the application of boundary conditions on the mesh.

Since every panel (in each particular dimensional space) has a similar characteristic form, the integration over each panel can be generalised. This function is carried out by the subroutines L2LC, L3LC, and L3ALC for the Laplace equation and H2LC, H3LC and H3ALC for the Helmholtz equation in two-, three- and axisymmetric three dimensional

[1] Boundary Value Problems and Boundary Conditions

problems respectively and these subroutines form the core modules for the Laplace or Helmholtz problems in the particular dimensional space.

The representation of the boundary by a set of characteristic panels enables us to easily define the boundary using data structures. For example an ordered list of the coordinates of the vertices of the approximating polygon in the figure above defines the boundary. The following panels show the simplest elements that can be used to approximate a surface.



The straight line, planar triangle and truncated cone panels.

Part of the numerical error in the boundary element solution will be a result of the approximation of the boundary. A better boundary approximation and a smaller numerical error will generally arise if the boundary is represented by curved panels. The methods described apply to only the simplest panels, straight line panels for two-dimensional boundaries[2], planar triangles for general three-dimensional boundaries[3] and truncated conical panels for axisymmetric problems[4].

[2] Representation of a line by straight line elements
[3] Representation of a surface by triangular panels
[4] Representation of an axisymmetric surface by conical panels